Performance Based Learning and Assessment Task

How Tall is the Tree?

I. ASSESSMENT TASK OVERVIEW & PURPOSE:
Students will use similar triangles and proportions to indirectly measure the height of a tree. They will determine in which direction(s) the tree could be cut down to create a standing area at the school's football field.

II. UNIT AUTHOR:
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III. COURSE:
Geometry

IV. CONTENT STRAND:
Geometry

V. OBJECTIVES:
Students will:
1) Understand how similar triangles are used to find indirect measurements
2) Apply the use of proportional reasoning

VI. REFERENCE/RESOURCE MATERIALS:
Students will use: pre-activity worksheet (attached), task instruction sheet (attached), calculator, pencil, toolbox of supplies (tape measure, mirror, string, protractor, cardboard), word processing software (i.e. Microsoft Word or Google Docs).

VII. PRIMARY ASSESSMENT STRATEGIES:
Students will be assessed on problem solving, communication, and mathematical representation.

VIII. EVALUATION CRITERIA:
Assessment will be based on the attached rubric. A benchmark of exemplary work is attached.

IX. INSTRUCTIONAL TIME:
One 90-minute period

How Tall is the Tree?

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Geometry

Mathematical Goals and Objective(s)
Students will understand how similar triangles are used to find indirect measurements. They will apply the use of proportional reasoning to solve problems.

Related SOL
SOL G.7 (The student, given information in the form of a figure or statement, will prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.)

NCTM Standards:

- Apply and adapt a variety of appropriate strategies to solve problems
- Explore relationships (including similarity) among classes of two-dimensional geometric objects, make and test conjectures about them, and solve problems involving them
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources
Students will use: pre-activity worksheet (attached), task instruction sheet (attached), calculator, pencil, toolbox of supplies (tape measure, mirror, string, protractor, cardboard), word processing software (i.e. Microsoft Word or Google Docs).

Assumption of Prior Knowledge
In order to be successful with this lesson, students should already know the definition of similar figures and the three ways to prove triangles are similar. Students should be working on at least the Analysis level on the Van Hiele scale with respect to similar figures. Students should be able to solve proportions. They may have difficulty setting up the correct proportion using the measurements from their diagrams. Students should know how to convert between inches and feet and between yards and feet.

Introduction: Setting Up the Mathematical Task

In this activity, students will use similar triangles and proportions to indirectly measure the height of a tree. The activity should take approximately 90 minutes.

The teacher will introduce the task with the following prompt: “Your principal and athletic director have enlisted your help to solve an issue with overcrowding in the stands at football games.”

The teacher should ask students to suggest ways to indirectly measure heights of tall objects. Ask the students to recall the definition of similar triangles and the ways to prove two triangles are similar.

If students only mention using the object's shadow, the teacher should ask them if there are ways to measure the height of objects without using the shadow.

Student Exploration

Individual Work
Students will first complete the pre-activity worksheet as a review of solving proportions.

Small Group Work
Students will be divided into pairs and given a task instruction sheet and toolbox of supplies. Each pair will measure the height of a tree around the school using two different methods of their choice. One student will take the measurements, and the other student will draw and label a diagram with the reported measurements. The pair of students will work together on setting up the proportions and solving them in order to find the height of the tree. They will compare the calculated heights from their two methods and discuss any discrepancies found. After performing their calculations, each group will determine the direction(s) the tree could be cut if it was located at the football field described on the task sheet. They will type a letter to the principal with their recommendation.

Student/Teacher Actions:
The teacher should walk around to the different groups and monitor that students are taking the correct measurements. Ask them why the triangles are similar. Address any mistakes seen in labeling the diagrams as well as setting up the correct proportions.

Monitoring Student Responses
After each group completes all parts of the task, they will share their findings in an oral report to the class. Students should complete the attached self-assessment.

The teacher will ask the students the following questions:

- When do we need to use the concept of similarity and indirect measurement? Why is it helpful?
- Why are similar figures proportional?

The teacher should also address with the class the common mistakes observed during the activity.

Assessment List and Benchmarks

See the attached rubric and student benchmark.

Journal/writing prompts
Give another real world example of using similar triangles to find indirect measurements.

Name _____

Pre-Activity Worksheet

Solve each proportion. Show all your work. Round to the nearest tenth, if necessary.

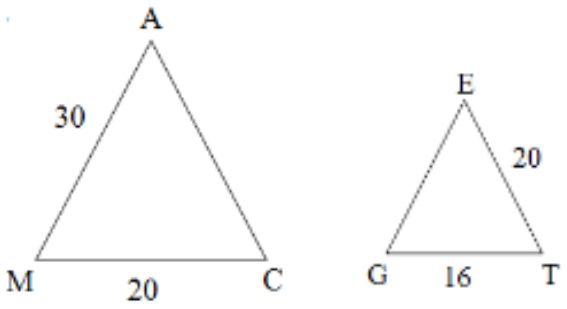
1. $\frac{2}{x} = \frac{6}{8}$

2. $\frac{x}{7} = \frac{2}{5}$

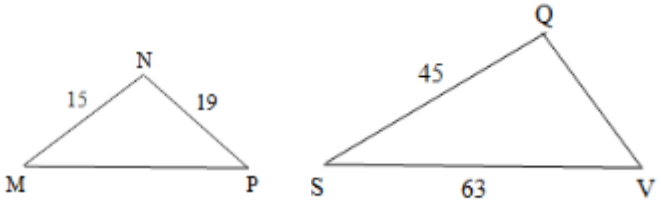
3. $\frac{5}{7} = \frac{x}{3}$

4. $\frac{3}{10} = \frac{2}{x}$

5. Find GE if $\Delta MAC \sim \Delta GET$.



6. Find MP if $\Delta MNP \sim \Delta SQV$.



Names _____

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How Tall is the Tree?

Your principal and athletic director have enlisted your geometry class to help solve an issue with overcrowding in the stands at football games. Since the bleachers are often so packed, many students stand around, and security continuously ask them to find seats that are, by and large, unavailable. Student Council suggests that a "standing area" for students be created in between the arc that links the home and away bleachers. However, there is a spruce tree in this area. If the tree is cut down, it may fall and hit surrounding structures. The entrance gate/ticket booth is located 40 yards north of the tree. The football field fence is 15 yards south of the tree. The field house is 30 yards east of the tree. The concession stand is located 25 yards west of the tree. Your task is to calculate the height of the tree using two different strategies and to determine in which direction(s) the tree could be cut without hitting any structure. You may use any of the supplies found in your toolbox. Write a letter to your principal with your recommendation.

Strategy 1: Give a brief description of your strategy. Draw and label a diagram and show your calculations.

Strategy 2: Give a brief description of your strategy. Draw and label a diagram and show your calculations.

Are your heights from Strategy 1 and Strategy 2 the same? If not, what do you think caused the discrepancies?

Assessment List

			Earned Assessment	
Number	**Element**	**Point Value**	**Self**	**Teacher**
1	Two different strategies are used to determine the tree's height.	2		
2	Diagrams are provided and correctly labeled.	2		
3	Calculations are provided and accurate.	2		
4	Letter of recommendation to principal is typed and clearly communicates student's thinking.	2		
5	Oral report to class uses precise mathematical language and clearly explains strategies and results.	2		
	Total	10		

Rubric

Number	Element	0	1	2
1	Two different strategies are used to determine the tree's height.	No strategy is used to determine the tree's height.	One strategy is used to determine the tree's height.	Two different strategies are used to determine the tree's height.
2	Diagrams are provided and correctly labeled.	Diagrams are not provided.	Diagrams are provided but labeled incorrectly.	Diagrams are provided and correctly labeled.
3	Calculations are provided and accurate.	Calculations are not shown.	Calculations are shown but inaccurate.	Calculations are shown and accurate.
4	Letter of recommendation to principal is typed and clearly communicates student's thinking.	Letter to principal is not complete.	Letter to principal is handwritten. Minimal communication and explanation	Letter to principal is typed and clearly communicates student's thinking. Explanation is thorough.
5	Oral report to class uses precise mathematical language and clearly explains strategies and results.	No oral report is given to the class.	Precise mathematical language is not used throughout the oral report. Strategies and results are not clearly explained.	Precise mathematical language is used throughout the oral report. Strategies and results are clearly explained.

Name KEY

Pre-Activity Worksheet

Solve each proportion. Show all your work. Round to the nearest tenth, if necessary.

1. $\frac{2}{x} = \frac{6}{8}$

$\frac{6x}{6} = \frac{16}{6}$

$x \approx 2.7$

2. $\frac{x}{7} = \frac{2}{5}$

$\frac{5x}{5} = \frac{14}{5}$

$x \approx 2.8$

3. $\frac{5}{7} = \frac{x}{3}$

$\frac{7x}{7} = \frac{15}{7}$

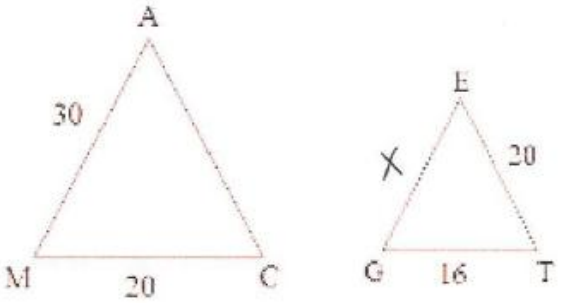
$x \approx 2.1$

4. $\frac{3}{10} = \frac{2}{x}$

$\frac{3x}{3} = \frac{20}{3}$

$x \approx 6.7$

5. Find GE if $\Delta MAC \sim \Delta GET$.

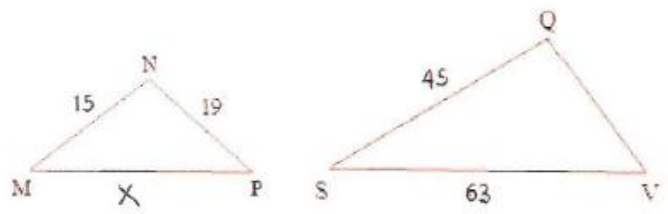


$\frac{x}{30} = \frac{16}{20}$

$\frac{20x}{20} = \frac{480}{20}$

$x = 24$

6. Find MP if $\Delta MNP \sim \Delta SQV$.



$\frac{x}{63} = \frac{15}{45}$

$\frac{45x}{45} = \frac{945}{45}$

$x = 21$

Names Benchmark

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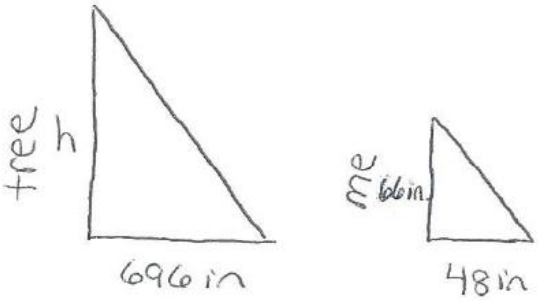
Strategy 1: Give a brief description of your strategy. Draw and label a diagram and show your calculations.

We measured my height, the length of my shadow, and the length of the tree's shadow. Then, we set up the similar triangles below.

height of tree = h
my height = 66 in.
tree's shadow = 696 in.
my shadow = 48 in.



$$\frac{h}{66} = \frac{696}{48}$$

$$\frac{48h}{48} = \frac{45936}{48}$$

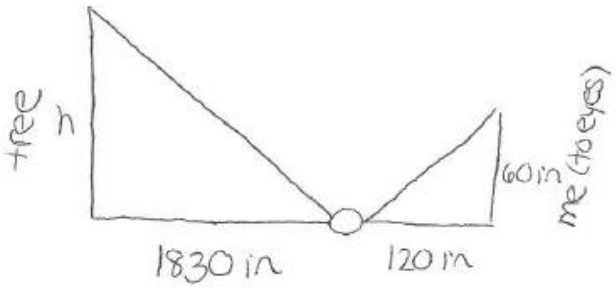
$$h = 957 \text{ in.}$$

$$957 \text{ in} \div 12 = \boxed{79.75 \text{ ft}}$$

Strategy 2: Give a brief description of your strategy. Draw and label a diagram and show your calculations.

We placed a mirror on the ground between me and the tree. I walked backwards until I could see the top of the tree in the mirror. My partner measured the distance between me and the mirror, the distance between the mirror and the tree, and the distance from my eyes down to the ground. We set up the similar triangles below.

height of tree = h
distance from my eyes to ground = 60 in.
distance between me and mirror = 120 in.
distance between tree and mirror = 1830 in.



$$\frac{h}{60} = \frac{1830}{120}$$

$$\frac{120h}{120} = \frac{109,800}{120}$$

$$h = 915 \text{ in}$$

$915 \text{ in} \div 12 = \boxed{76.25 \text{ ft}}$

Are your heights from Strategy 1 and Strategy 2 the same? If not, what do you think caused the discrepancies?

The heights from Strategy 1 and Strategy 2 are not equal. The discrepancy could be caused by inaccurate measuring with the tape measure.

Benchmark (Sample Student Letter)

December 7, 2015

Dear Mr. Long:

Thank you for asking our geometry class to assist you with the issue of cutting down the spruce tree at the football field to create more space for students. We were given the task of determining in which direction(s) the tree could be cut without hitting any structure. Using two different strategies with similar triangles, our group calculated the height of the spruce tree to be between approximately 76.25 feet and 79.75 feet.

Based on these calculations, we recommend that the tree be cut so that it will fall to its north or to its east. Since the tree is shorter than its distance from the entrance gate/ticket booth (40 yards or 40 x 3 = 120 feet) and field house (30 yards or 30 x 3 = 90 feet), cutting the tree in those directions would not result in hitting either structure.

Please let us know if you have any questions about our calculations.

Sincerely,

Jane Smith & Matt Jones